QUANTUM FIELD THEORY IN A NUTSHELL

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Since $\nu = 1/k$, we have here the classic Laughlin odd-denominator Hall fluids with filling factor $\nu = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \cdots$. The famous result that the quasiparticles carry fractional charge and statistics just pops out [see (9) and (11)].

This is truly dramatic: a bunch of electrons moving around in a plane with a magnetic field corresponding to $\nu = \frac{1}{3}$, and lo and behold, each electron has fragmented into three pieces, each piece with charge $\frac{1}{3}$ and fractional statistics $\frac{1}{3}$!

A new kind of order

The goal of condensed matter physics is to understand the various states of matter. States of matter are characterized by the presence (or absence) of order: a ferromagnet becomes ordered below the transition temperature. In the Landau-Ginzburg theory, as we saw in Chapter V.3, order is associated with spontaneous symmetry breaking, described naturally with group theory. Girvin and MacDonald first noted that the order in Hall fluids does not really fit into the Landau-Ginzburg scheme: We have not broken any obvious symmetry. The topological property of the Hall fluids provides a clue to what is going on. As explained in the preceding chapter, the ground state degeneracy of a Hall fluid depends on the topology of the manifold it lives on, a dependence group theory is incapable of accounting for. Wen has forcefully emphasized that the study of topological order, or more generally quantum order, may open up a vast new vista on the possible states of matter.

Comments and generalization

Let me conclude with several comments that might spur you to explore the wealth of literature on the Hall fluid.

1. The appearance of integers implies that our result is robust. A slick argument can be made based on the remark in the previous chapter that the Chern-Simons term does not know about clocks and rulers and hence can't possibly depend on microphysics such as the scattering of electrons off impurities which cannot be defined without clocks and rulers. In contrast, the physics that is not part of the topological field theory and described by $(\cdots)$ in (3) would certainly depend on detailed microphysics.

2. If we had followed the long way to derive the effective field theory of the Hall fluid, we would have seen that the quasiparticle is actually a vortex constructed (as in Chapter V.7) out of the scalar field representing the electron. Given that the Hall fluid is incompressible, just about the only excitation you can think of is a vortex with electrons coherently whirling around.

3. In the previous chapter we remarked that the Chern-Simons term is gauge invariant only upon dropping a boundary term. But real Hall fluids in the laboratory live in samples with boundaries. So how can (3) be correct? Remarkably, this apparent "defect" of the theory actually represents one of its virtues! Suppose the theory (3) is defined on a bounded 2-dimensional manifold, a disk for example.